Exponential Function Applications

Formula: $y = a \cdot b^x$

Monday - 2,000 bacteria
Wednesday - 4,500 bacteria

a) By what factor did the bacteria increase?

$\frac{4500}{2000} = 2.25$ every 2 days

Friday $= 4{,}500 \times 2.25 = 10{,}125$ bacteria
Sunday $= 10{,}125 \times 2.25 = 22{,}782$ bacteria

b) started Friday

$y = a \cdot b^x$

x	f(x)
3	2,000
5	4,500

$\frac{4500 = a \cdot b^5}{2000 = a \cdot b^3}$

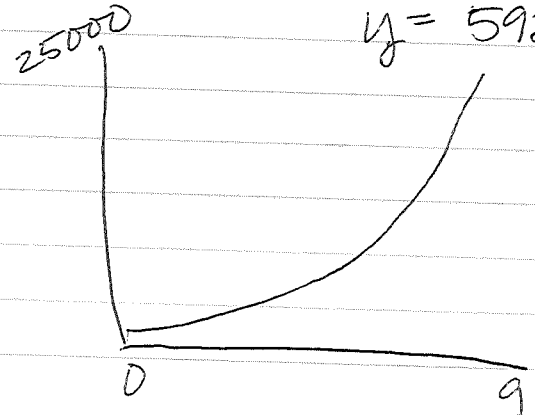
$\sqrt{2.25} = \sqrt{b^2}$

$1.5 = b$

$\frac{4500 = a(1.5^5)}{1.5^5 \quad 1.5^5}$

$592.593 = a$

$y = 592.593(1.5^x)$



Trace 3 → 2000
Trace 5 → 4500

c) Predict # of bacteria on Thursday
Trace 6 → 6,750 bacteria
extrapolation - data only goes to Wednesday

d) When will the # of bacteria = 40,000?
$Y_2 = 40{,}000$
calc → int **day 10**

e) What does dilation factor tell about problem?

$$y = 593(1.5)^x$$

593 → starting # of bacteria

Solve Exponential Equations

✓ $4x - 7 = 19$

✓ $2x^2 - 5x + 2$

Today: $3^x = 5$ → solved w/ logarithms

$\log 3^x = \log 5$

$x \log 3 = \log 5$

$\frac{x \log 3}{\log 3} = \frac{\log 5}{\log 3}$

$x = 1.465$

Ex. $\frac{18{,}000(.85)^x}{18{,}000} = \frac{2{,}560}{18{,}000}$

$.85^x = .14\overline{2}$

$\log .85^x = \log(.14\overline{2})$

$\frac{x \log(.85)}{\log(.85)} = \frac{\log(.14\overline{2})}{\log(.85)}$

$\boxed{x = 12{,}001}$

Ex. If $f(x) = 34 \cdot 3^x$, find x algebraically if $f(x) = 1{,}234$. Check numerically.

$\frac{34 \cdot 3^x}{34} = \frac{1234}{34}$

$3^x = \frac{617}{17}$

$\log 3^x = \log\left(\frac{617}{17}\right)$

$\frac{x \log 3}{\log 3} = \frac{\log\left(\frac{617}{17}\right)}{\log 3}$

$\boxed{x = 3{,}269}$

Check:

$34 \cdot 3^{3.269} \stackrel{?}{=} 1234$

$1234 = 1234$ ✓

Ex. Show numerically that $\log(5 \cdot 8) = \log 5 + \log 8$

$\log_{10} 10^y = y$

$1.6020\ldots = .6989 + .9030$

$1.6020 = 1.602$

re-write $\log_{10} 2 = y$ "log base 10 of 2 equals y"

$10^y = 2$ "10 to the power of y = 2"

Change of Base Property

$$\log_a b = \frac{\log b}{\log a}$$ (log of argument) / (log of base)

Ex. Find $\log_3 57$ using change of base

$$\log_3 57 = \frac{\log 57}{\log 3}$$

Ex. Find x, if $\log_x 4 = \frac{2}{3}$

$$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = (4)^{\frac{3}{2}}$$

$x^{\frac{2}{3}\cdot\frac{3}{2}}$

$$x^1 = \left(\sqrt{4}\right)^3$$

$$x = 2^3$$

$$\boxed{x = 8}$$