**GEOMETRY EOC Study Guide**

**Spring 2012**

**PART I:**

**Essentials of Geometry:**

**Undefined terms** (do not have formal definitions): Points, Lines, and Planes

|  |  |
| --- | --- |
| **Term and definition** | **Picture and notation** |
| **Point -**  |  |
| **Line -**  |  |
| **Plane -**  |  |

**Collinear** points – points that lie on the same line.

**Coplanar points** – points that lie in the same plane.

**Defined terms** (can be described using known words such as point or line): line segment, segment, endpoints, ray.

|  |  |
| --- | --- |
| **Term and definition** | **Picture and notation** |
| **Line Segment-**  |  |
| **Segment –** **Endpoints -**  |  |
| **Ray -**  |  |

**Opposite rays** – If point C lies on line AB between A and B, then ray CA and ray CB are opposite rays.

Where do planes **intersect**? Sketch a picture.

Where do lines **intersect**? Sketch a picture.

Where does a line and a plane **intersect**? Sketch a picture.

**Using segment postulates to identify congruent segments:**

**Define:**

**Postulate –**

**Axiom –**

**Theorem –**

**Corollary to a theorem –**

**Congruent segments –**

|  |  |
| --- | --- |
| **RULER POSTULATE:**  | **PICTURE:** |

**Midpoint and Distance in the Coordinate Plane:**

**Define the following:**

**Midpoint –**

**Segment bisector –**

|  |  |  |
| --- | --- | --- |
| **Midpoint in the Coordinate Plane** | Formula: | Example: |
| **Distance in the Coordinate Plane** | Formula:  | Example: |

**Measuring and Classifying Angles:**

Label the angle, sides, and vertex of the angle.

Classifying Angles – draw an example and label an acute, right, obtuse, and straight angle.

|  |  |
| --- | --- |
| **PROTRACTOR POSTULATE:**  | **PICTURE:** |
| **ANGLE ADDITION POSTULATE:** | **PICTURE:** |

Define and draw an example of the following terms:

**Congruent angles –**

**Angle bisector –**

A **Construction** is a geometric drawing that uses a limited set of tools, usually a compass and straightedge. Construct the following and show your steps:

**Copy a segment:**

**Bisect a segment:**

**Copy an angle:**

**Bisect an angle:**

**Angle Pair Relationships:**

**Define the following:**

**Complementary angles –**

**Supplementary angles –**

**Adjacent angles –**

**Vertical angles –**

**Linear Pair –**

**Classifying Polygons:**

Define the following and draw a picture of each:

**Polygon –**

**Sides –**

**Vertex of a polygon –**

**Convex –**

**Concave –**

**Equilateral polygon –**

**Equiangular polygon –**

**Regular polygon –**

**Finding Perimeter, Circumference, and Area:**

Write the formulas for Perimeter, Area, and Circumference for the following figures. Draw a picture and give an example of each.

|  |  |
| --- | --- |
| **Square:** | **Rectangle:** |
| **Triangle:** | **Circle:** |

Part II

**Using Inductive Reasoning:**

Define the following terms and answer the questions pertaining to each term:

**Conjecture –**

Given five collinear points, make a conjecture about the number of ways to connect different pairs of the points.

**Draw a picture:**

**Conjecture:**

**Inductive reasoning –**

**Counterexample –**

A student makes the following conjecture about the sum of two numbers. Find a counterexample to disprove the student’s conjecture.

**Conjecture:** The sum of two numbers is always greater than the larger number.

**Solution and counterexample:**

Based on your solution, is your original conjecture TRUE or FALSE?

**Analyzing Conditional Statements:**

Define the following terms:

**Conditional statement –**

**Hypothesis –**

**Conclusion –**

**Negation –**

**Converse –**

**Inverse –**

**Contrapositive –**

**Equivalent Statements –**

**Biconditional statements –**

Label the hypothesis and the conclusion of the following conditional statement. Then write the negation, converse, inverse, and contrapositive of it and determine the truth value of each. Determine if any pairs of statements are equivalent statements.

**If it is raining, then there are clouds in the sky.**

Write the definition of perpendicular lines as a biconditional statement below.

**Applying Deductive Reasoning:**

**Define deductive reasoning –**

Define and draw an example of each:

|  |  |
| --- | --- |
| **Law of Detachment -**  | **Example –**  |
| **Law of Syllogism –**  | **Example –**  |

**Using Postulates and Diagrams:**

Remember: A line is a **line perpendicular to a plane** IF and ONLY IF the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.

**Reason Using Properties from Algebra:**

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| --- | --- |
| **Algebraic Properties of Equality** | **Algebraic Example** |
| **Addition Property** |  |
| **Subtraction Property** |  |
| **Multiplication Property** |  |
| **Division Property** |  |
| **Substitution Property** |  |

Solve the following algebraic equation and show each step. Label each step with the property modeled.

-4(11*x* + 2) = 80

Properties of equality that pertain to segment lengths and angle measures:

**Reflexive Property of Equality**

|  |  |
| --- | --- |
| **Real Numbers** |  |
| **Segment Length** |  |
| **Angle Measure** |  |

**Symmetric Property of Equality**

|  |  |
| --- | --- |
| **Real Numbers** |  |
| **Segment Length** |  |
| **Angle Measure** |  |

**Transitive Property of Equality**

|  |  |
| --- | --- |
| **Real Numbers** |  |
| **Segment Length** |  |
| **Angle Measure** |  |

**Prove Statements about Segments and Angles:**

Define the following:

**Proof –**

**Two-column proof –**

Define the following theorems and show how they are reflexive, symmetric, and transitive.

**Congruence of Segments Theorem –**

|  |  |
| --- | --- |
| **Reflexive** |  |
| **Symmetric** |  |
| **Transitive** |  |

**Congruence of Angles Theorem –**

|  |  |
| --- | --- |
| **Reflexive** |  |
| **Symmetric** |  |
| **Transitive** |  |

**Proving Angle Pair Relationships:**

Define the following theorems.

|  |  |
| --- | --- |
| **RIGHT ANGLES CONGRUENCE THEOREM** |  |
| **CONGRUENT SUPPLEMENTS THEOREM** |  |
| **CONGRUENT COMPLEMENTS THEOREM** |  |
| **LINEAR PAIR POSTULATE** |  |
| **VERTICAL ANGLES CONGRUENCE THEOREM** |  |

Prove the right angles theorem in the two-column proof below.

**Given:** Two angles (angle 1 and angle 2). < 1 and < 2 are right angles.

**Prove:** $<1 ≅ <2$

|  |  |
| --- | --- |
| **Statements** | **Reasons** |
|  |  |
|  |  |
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**Triangles:**

In your study guide, you should have ALL the information on triangles, including:

* How to classify triangles by measures of angles and lengths of sides
* Triangle Theorems, converse theorems, corollaries
* Midsegments, perpendicular bisectors, angle bisectors, altitudes, and medians
* How to prove triangles congruent
* How to prove triangles similar (postulates and theorems), similarity ratios, scale factor, similar polygons, similarity statements
* Triangle proportionality theorem
* Dilations
* Incenter, centroid, orthocenter
* Pythagorean theorem (and its converse theorem), Pythagorean triple

In addition to your study guide notes, you will need to know the following about triangles:

|  |  |
| --- | --- |
| If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side. | Example on p. 328 |
| If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle. | Example on p. 328 |
| **Triangle Inequality Theorem** – The sum of the lengths of any two sides of a triangle is greater than the length of the third side. | Example on p. 330 |
| **Hinge Theorem** – If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second. | Example on p. 335 |
| **Converse of the Hinge Theorem –** If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second. | Example on p. 335 |
| If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.  | Example p. 449 |
| **Geometric Mean** – In right triangle ABC, altitude CD forms two smaller triangles so that triangle CBD is similar to triangle ACD is similar to triangle ABC. | Example p. 493 |
| **45-45-90 Triangle Theorem** – In a 45-45-90 triangle, the hypotenuse is $√2$ times as long as each leg.Hypotenuse = leg $∙ √2$ | Example p. 457 |
| **30-60-90 Triangle Theorem –** In a 30-60-90 triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $√3$ times as long as the shorter leg. Hypotenuse = 2 ∙ shorter legLonger leg = shorter leg $∙ √3$ | Example p. 459 |

**Applying trigonometric ratios:**

Define the following terms:

**Trigonometric ratio –**

**Tangent –**

**Sine –**

**Cosine –**

**Angle of Elevation –**

**Angle of Depression –**

|  |  |
| --- | --- |
| **Tangent Ratio –**  | Example p. 466 |
| **Example p. 467 (Tangent ratio)** | Solution: |
| **Sine Ratio –**  | Example p. 473 |
| **Example p. 473 (Sine ratio)** | Solution: |

|  |  |
| --- | --- |
| **Cosine Ratio –**  | Example p. 473 |
| **Example p. 474 (Cosine ratio)** | Solution: |

**Solving right triangles:**

Use inverse trigonometric ratios:

Triangle ABC is a right triangle. Angle C is 90 degrees and angle A is an acute triangle. Draw the triangle below (actual triangle on p. 483).

|  |  |  |
| --- | --- | --- |
| **Trig Ratio** | **Conditional Statement** | **Algebra Model** |
| **Inverse Tangent** |  |  |
| **Inverse Sine** |  |  |
| **Inverse Cosine** |  |  |

Angle Measures in Polygons:

Define the following terms:

Diagonal –

|  |  |  |
| --- | --- | --- |
| **Theorem** | **Picture** | **Conclusion**  |
| **Polygon Interior Angles Theorem** |  | The sum of the measures of the interior angles of a convex n-gon is (n – 2) ∙ 180. |
| **Interior Angles of a Quadrilateral** |  | The sum of the measures of the interior angles of a quadrilateral is 360.  |
| **Polygon Exterior Angles Theorem** |  | The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360. |

Parallelogram –

**Properties of Parallelograms**

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| --- | --- | --- |
| **Theorem** | **Hypothesis** | **Conclusion / Picture** |
| If a quadrilateral is a parallelogram, then its opposite sides are congruent. |  |  |
| If a quadrilateral is a parallelogram, then its opposite angles are congruent. |  |  |
| If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. |  |  |
| If a quadrilateral is a parallelogram, then its diagonals bisect eachother. |  |  |

Don’t forget – you can use the **converse** of the 1st, 2nd, and 4th conditional statements above to **prove a quadrilateral is a parallelogram**! You can also use:

|  |  |  |
| --- | --- | --- |
| If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram. |  |  |

**Rhombus –**

**Rectangle –**

**Square –**

**Corollaries**

|  |  |  |
| --- | --- | --- |
| **Corollary** | **Description** | **Picture / Example** |
| **Rhombus Corollary** |  |  |
| **Rectangle Corollary** |  |  |
| **Square Corollary** |  |  |

**Theorems**

|  |  |  |
| --- | --- | --- |
| **Theorem** | **Description with notation** | **Example / Picture** |
| A parallelogram is a rhombus if and only if its diagonals are perpendicular. |  |  |
| A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles. |  |  |
| A parallelogram is a rectangle if and only if its diagonals are congruent. |  |  |

Trapezoid (draw a picture at right and label ALL parts) –

Isosceles Trapezoid (draw a picture at right and label) –

|  |  |  |
| --- | --- | --- |
| **Theorem** | **Hypothesis** | **Conclusion / Picture** |
| If a trapezoid is isosceles, then each pair of base angles is congruent |  |  |
| If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid. |  |  |
| A trapezoid is isosceles if and only if its diagonals are congruent. |  |  |
| **Midsegment Theorem for Trapezoids –** |  |  |

Kite –

|  |  |  |
| --- | --- | --- |
| **Theorem** | **Description in notation** | **Example / Picture** |
| A quadrilateral is a kite, then its diagonals are perpendicular.  |  |  |
| If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent. |  |  |

**Transformations:**

In your study guide, you should have ALL the information on transformations, including:

* Image, preimage, isometry (a transformation that preserves length and angle measure; i.e. congruence transformation)
* Translation theorem
* Line of reflection, Coordinate rules for reflections
* Mapping notations
* Center of rotation, angle of rotation, coordinate rules for rotations about the Origin
* Composition of transformations, glide reflections
* Reflections in parallel lines theorem, Reflections in intersecting lines theorem
* Line of symmetry, center of symmetry, rotational symmetry

**Circles:**

In your study guide, you should have ALL the information on circles, including:

* Circle, center, radius, chord, diameter, secant, tangent
* Theorem – tangent line is perpendicular to radius
* Theorem – tangent segments with common external point as vertex are congruent
* Central angle, inscribed angle, minor arc, major arc, semicircle, measure of a major/minor arc
* Congruent circles, congruent arcs, Arc addition postulate
* Arc length corollary, Area of a sector
* Theorems involving congruent chords/arcs, perpendicular bisecting chords, perpendicular bisector chords to diameters
* Measure of an inscribed angle theorem, inscribed angles with same intercepted arc are congruent
* Hypotenuse of a right triangle inscribed in a circle is the diameter
* Quadrilateral inscribed in a circle will have opposite angles supplementary
* Angles inside the circle theorem
* Angles outside the circle theorem
* Tangent and chord intersect at point of tangency – angle formed is half measure of intercepted arc (theorem)
* Segments of chords theorem, segments of secants theorem, segments of secants and tangents theorem
* Write standard equation of a circle

**Perimeters and Areas of 2-D figures:**

In your study guide, you should have ALL the information on perimeter/area of 2-D figures, including:

* Know how to calculate area and perimeter of triangles, parallelograms, trapezoids and circumference and area of a circle
* Areas of similar polygons theorem
* Area of a REGULAR polygon (p. 763), apothem

**Geometric Probability**

Define the following terms:

**Probability –**

**Geometric Probability –**

Give an example of finding probability in terms of length (p. 771)

Give an example of finding probability in terms of area (p. 772)