Matrices

Vocabulary:

Matrix –

Scalar –

Matrix A has two rows and three columns. A matrix with m rows and n columns has **dimensions** m x n, read "m by n," and is called an m x n matrix. Each value in a matrix is called an **entry** of the matrix.

$$A = \begin{bmatrix} 16.781 & 16.29 & 17.318 \\ 16.206 & 16.606 & 17.668 \end{bmatrix} \begin{matrix} \leftarrow \text{Row 1} \\ \leftarrow \text{Row 2} \end{matrix}$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ \text{Column 1} & \text{Column 2} & \text{Column 3} \end{matrix}$$

The address of an entry is its location in a matrix, expressed using the lowercase matrix letter with the row and column number as subscripts. The score 16.206 is located in row 2 column 1, so a_{21} is 16.206.

$$A = \begin{bmatrix} 16.781 & 16.29 & 17.318 \\ 16.206 & 16.606 & 17.668 \end{bmatrix}$$

$a_{21} \nearrow$

Displaying Data is matrix form

Cost of 4-Inch Cubic Box ($)		
	Plastic	Paper
Total Cost	0.48	0.72
Cost per in^2	0.005	0.0075
Cost per in^3	0.0075	0.01125

Use the packaging data for the costs of the packages given.

a. Display the data in matrix form.

b. What are the dimensions of C?

Adding and Subtracting Matrices

Know it! Note

Adding and Subtracting Matrices

WORDS	NUMBERS	ALGEBRA
To add or subtract two matrices, add or subtract the corresponding entries.	$\begin{bmatrix} 1 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 10 \end{bmatrix} = \begin{bmatrix} 6 & 12 \end{bmatrix}$	$\begin{bmatrix} a_{11} & a_{12} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \end{bmatrix} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \end{bmatrix}$

You can add or subtract two matrices only if they have the same dimensions.

✔ Same Dimensions	✘ Different Dimensions
$\begin{bmatrix} 1 & 2 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 7 & 6 \end{bmatrix}$ $\quad \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} + \begin{bmatrix} 2 \\ 8 \\ 1 \end{bmatrix}$	~~$\begin{bmatrix} 1 & 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 10 \end{bmatrix}$~~ $\quad$ ~~$\begin{bmatrix} a_{11} & a_{12} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \end{bmatrix}$~~

Add or subtract, if possible.

Finding Matrix Sums and Differences

$$A=\begin{bmatrix} 4 & -2 \\ -3 & 10 \\ 2 & 6 \end{bmatrix} \quad B=\begin{bmatrix} 4 & -1 & -5 \\ 3 & 2 & 8 \end{bmatrix} \quad C=\begin{bmatrix} 3 & 2 \\ 0 & -9 \\ -5 & 14 \end{bmatrix} \quad D=\begin{bmatrix} 0 & 1 & -3 \\ 3 & 0 & 10 \end{bmatrix}$$

A $A+C$

Add each corresponding entry.

$$A+C=\begin{bmatrix} 4 & -2 \\ -3 & 10 \\ 2 & 6 \end{bmatrix}+\begin{bmatrix} 3 & 2 \\ 0 & -9 \\ -5 & 14 \end{bmatrix}=$$

B $C-A$

Subtract each corresponding entry.

$$C-A=\begin{bmatrix} 3 & 2 \\ 0 & -9 \\ -5 & 14 \end{bmatrix}-\begin{bmatrix} 4 & -2 \\ -3 & 10 \\ 2 & 6 \end{bmatrix}=$$

C $C+B$

Multiply a Matrix by a scalar

To find the product of a scalar and a matrix, or the *scalar product*, multiply each entry by the scalar.

Example 1:

$$2\begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}=$$

Example 2: ***Business Application***

A ticket service marks up prices on tickets to rodeos and other events by 150%. Use a scalar product to find the marked-up prices.

Rodeo Ticket Prices		
Days	Plaza	Balcony
1–2	\$60	\$35
3–8	\$50	\$28
9–10	\$80	\$45

Ticket Service Prices		
Days	Plaza	Balcony
1–2	\$150	\$87.50
3–8	\$125	\$70.00
9–10	\$200	\$112.50

You can multiply by 1.5 and add to the original numbers.

$$\begin{bmatrix} 60 & 35 \\ 50 & 28 \\ 80 & 45 \end{bmatrix}+1.5\begin{bmatrix} 60 & 35 \\ 50 & 28 \\ 80 & 45 \end{bmatrix}$$

Simplifying Matrix Expressions

$A = \begin{bmatrix} 4 & -2 \\ -3 & 10 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -1 & -5 \\ 3 & 2 & 8 \end{bmatrix}$ $C = \begin{bmatrix} 3 & 2 \\ 0 & -9 \end{bmatrix}$ $D = \begin{bmatrix} -6 & 3 & 8 \end{bmatrix}$

A Evaluate $2A - 3B$, if possible.

$2\begin{bmatrix} 4 & -2 \\ -3 & 10 \end{bmatrix} - 3\begin{bmatrix} 4 & -1 & -5 \\ 3 & 2 & 8 \end{bmatrix}$

Do A and B have the same dimensions? Can they be subtracted after the scalar products are found?

B Evaluate $C - 2A$, if possible.

$= \begin{bmatrix} 3 & 2 \\ 0 & -9 \end{bmatrix} - 2\begin{bmatrix} 4 & -2 \\ -3 & 10 \end{bmatrix}$

Multiplying Matrices

Matrix Product –

The following rules apply when multiplying matrices.

- Matrices A and B can be multiplied only if the number of columns in A equals the number of rows in B.
- The product of an $m \times n$ and an $n \times p$ matrix is an $m \times p$ matrix.

$A = \begin{bmatrix} 3 & 5 & 7 \\ 4 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 & 3 & 8 \\ 9 & 5 & 2 & 0 \\ 0 & 1 & 6 & 7 \end{bmatrix}$

A B AB
$2 \times 3 \quad 3 \times 4 = 2 \times 4$ matrix
columns = rows

$C = \begin{bmatrix} 3 & 5 \\ 4 & 1 \\ 5 & 8 \end{bmatrix}$ $D = \begin{bmatrix} 2 & 3 & 3 & 8 & 4 \\ 9 & 5 & 2 & 0 & 6 \\ 0 & 1 & 6 & 7 & 2 \end{bmatrix}$

C D
$3 \times 2 \quad 3 \times 5$
columns $\neq$ rows

✗ CD is not defined $(2 \neq 3)$

Identifying Matrix Products

Tell whether each product is defined. If so, give its dimensions.

A $P_{2 \times 5}$ and $Q_{5 \times 3}$; PQ

P Q PQ
$2 \times 5 \quad 5 \times 3 = 2 \times 3$ matrix

The inner dimensions are equal $(5 = 5)$, so the matrix product is defined. The dimensions of the product are the outer numbers, 2×3.

B $R_{4 \times 3}$ and $S_{4 \times 5}$; RS

R S
$4 \times 3 \quad 4 \times 5$

The inner dimensions are not equal $(3 \neq 4)$, so the matrix product is not defined. ✗

Use the matrices in Example 1. Tell whether each product is defined. If so, give its dimensions.

1a. QP

1b. SR

1c. SQ

Know it! Note

Multiplying Matrices

WORDS	NUMBERS	ALGEBRA
In a matrix product $P = AB$, each element p_{ij} is the sum of the products of consecutive entries in row i in matrix A and column j in matrix B.	$P = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1 \cdot 5 + 2 \cdot 7 & 1 \cdot 6 + 2 \cdot 8 \\ 3 \cdot 5 + 4 \cdot 7 & 3 \cdot 6 + 4 \cdot 8 \end{bmatrix}$	$P = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}\begin{bmatrix} c_1 & c_2 \\ d_1 & d_2 \end{bmatrix} = \begin{bmatrix} a_1c_1 + a_2d_1 & a_1c_2 + a_2d_2 \\ b_1c_1 + b_2d_1 & b_1c_2 + b_2d_2 \end{bmatrix}$

Finding the Matrix Product

Find each product, if possible.

$$A = \begin{bmatrix} 0 & 4 & 9 \\ -3 & 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 1 \\ -2 & 7 \\ 6 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 11 & -1 \\ 12 & 10 \end{bmatrix}$$

AB

Check the dimensions. A is 2 x 3, B is 3 x 2. *AB* is defined and is 2 x 2. Multiply row 1 of A and column 1 of B. Place the result in ab_{11}.

Step 1) Multiply row 1 of A and column 2 of B. Place the result in ab_{12}.

Step 2) Multiply row 2 of A and column 1 of B. Place the result in ab_{21}.

Step 3) Multiply row 2 of A and column 2 of B. Place the result in ab_{22}.

Now, multiply *BA*.

Try *AC*.

BC.

A **square matrix** is any matrix that has the same number of rows as columns; it is an n x n matrix. The **main diagonal** of a square matrix is the diagonal from the upper left corner to the lower right corner.

The **multiplicative identity matrix** is any square matrix, named with the letter I, that has all of the entries along the main diagonal equal to 1 and all the other entries equal to 0.

$$I_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix I is the multiplicative identity when A is any square matrix and $AI = IA = A$.

For $A = \begin{bmatrix} 5 & 7 \\ -1 & 4 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and

$AI = \begin{bmatrix} 5 & 7 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} =$

$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & 7 \\ -1 & 4 \end{bmatrix} =$

Finding Powers of Square Matrices

$$A = \begin{bmatrix} 7 & 3 \\ -2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 4 & 1 \\ 5 & 0 & -2 \\ 1 & -1 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & -2 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

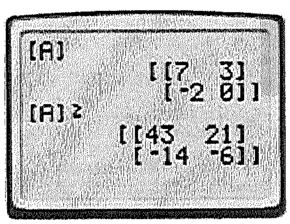
Evaluate, if possible.

A A^2

$$A^2 = \begin{bmatrix} 7 & 3 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 7 & 3 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 7(7) + 3(-2) & 7(3) + 3(0) \\ -2(7) + 0(-2) & -2(3) + 0(0) \end{bmatrix}$$

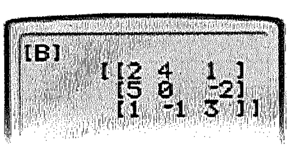
$$= \begin{bmatrix} 43 & 21 \\ -14 & -6 \end{bmatrix}$$

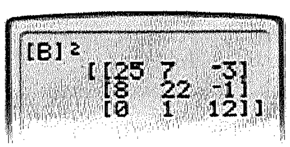


Check Use a calculator.

B B^2

For large matrices, use a graphing calculator.





CHECK IT OUT! **Evaluate, if possible.**

4a. C^2 **4b.** A^3 **4c.** B^3 **4d.** I^4