

# Key

## Notes: Using the Definition and Properties of Logarithms to Solve Equations

Type I - Logarithms on both sides of the equation

- Condense each side to a single log
- As long as the bases on each side are the same they will cancel out
- Solve the remaining equation
- Answer may not create the log of a negative number!

Example:  $\log_4 24 - \log_4 x = \log_4 x + \log_4 6$

$$\cancel{\log_4} \left( \frac{24}{x} \right) = \cancel{\log_4} (x \cdot 6) \quad (\text{condense + cross off logs})$$

$$\frac{24}{x} = \frac{6x}{1} \quad (\text{cross-multiply})$$

$$\frac{6x}{6} = \frac{24}{6}$$

$$x^2 = 4$$

$$x = \pm 2$$

(NOTE - You can't have the log of a negative number, so "throw out" the negative 2)

Final answer:  
 $x = 2$

Example:  $\log_6(x+2) + \log_6(x+4) = \log_6 3$

$$\cancel{\log_6} ((x+2)(x+4)) = \cancel{\log_6} 3 \quad (\text{Condense})$$

(Cross off logs)

$$x^2 + 4x + 2x + 8 = 3$$

$$x^2 + 6x + 8 = 3$$

$$x^2 + 6x + 5 = 0$$

$$(x+5)(x+1) = 0$$

$$x+5=0 \quad x+1=0$$

$$x = -5$$

$$x = -1$$

Check:  $\log_6(-5+2) =$

$$\cancel{\log_6 3}$$

-5 is not a valid answer

$$\log_6(-1+2) = \log_6 1 \quad -$$

$$\log_6(-1+4) = \log_6 3 \quad -$$

Type 2 - Logarithms on only one side of the equation

- Condense the one side into a single log
- Change to exponential form
- Solve the equation

Example:  $\log 20 + \log x - \log 2 = .6180$

$$\log\left(\frac{20x}{2}\right) = .6180$$

$$\log 10x = .6180 \quad (\log \text{ w/o a base} = \log \text{ base } \underline{10})$$

$$\log_{10}(10x) = .6180 \quad (\text{Change to exponential form})$$

$$10^{.6180} = 10x$$

$$4.1495 = \frac{10x}{10}$$

$$x = .4150 \text{ (rounded)}$$

Example:  $\log x + \log 8 = 3$

$$\log(8x) = 3$$

$$\log_{10}(8x) = 3$$

$$10^3 = 8x$$

$$\frac{1000}{8} = \frac{8x}{8}$$

$$x = 125$$

Example 3:  $\log_2(9x+5) - \log_2(x^2-1) = 2$

$$\log_2\left(\frac{9x+5}{x^2-1}\right) = 2 \quad (\text{Change to exponential form})$$

$$\frac{2^2}{1} = \frac{9x+5}{x^2-1} \quad (\text{Cross multiply})$$

$$4(x^2-1) = 9x+5$$

$$4x^2-4 = 9x+5$$

$$4x^2-9x-9 = 0$$

$$x^2-9x-36 = 0$$

$$(x-\frac{12}{4})(x+\frac{3}{4}) = 0$$

$$(x-3)(4x+3) = 0$$

$$x=3 \quad x=\frac{-3}{4}$$