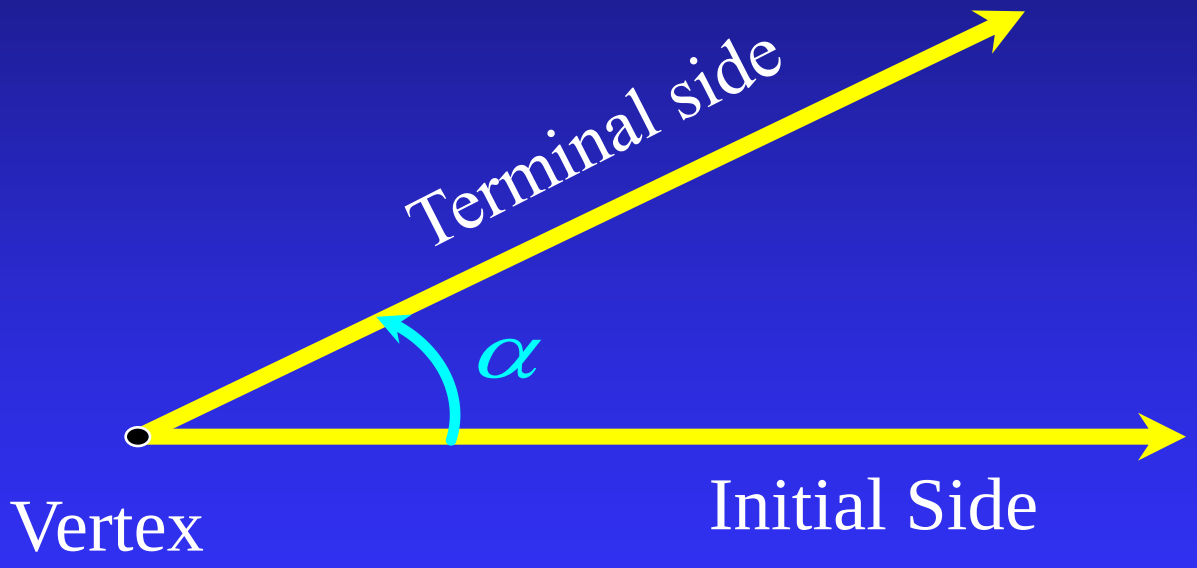
Trigonometry: Angles

Objectives of this Presentation:

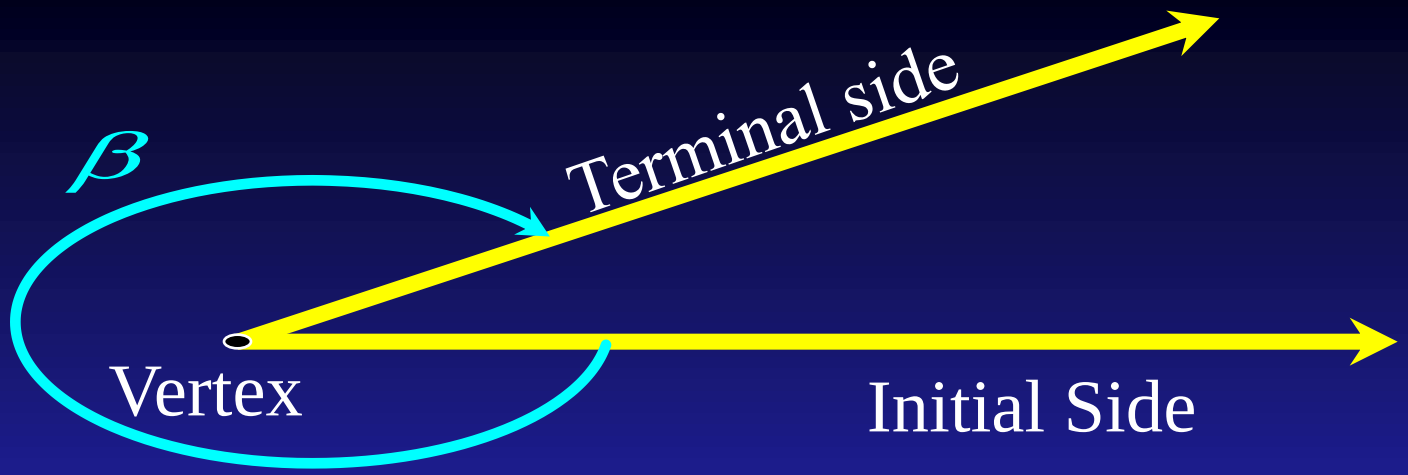
- Build basic definitions of angles as built on the coordinate plane.
- Learn how to measure both positive & negative angles
- Convert Between Degrees, Minutes, Seconds, and Decimal Forms for Angles
- Convert From Degrees to Radians, Radians to Degrees

A **ray**, or **half-line**, is that portion of a line that starts at a point *V* on the line and extends indefinitely in one direction. The starting point *V* of a ray is called its **vertex**.

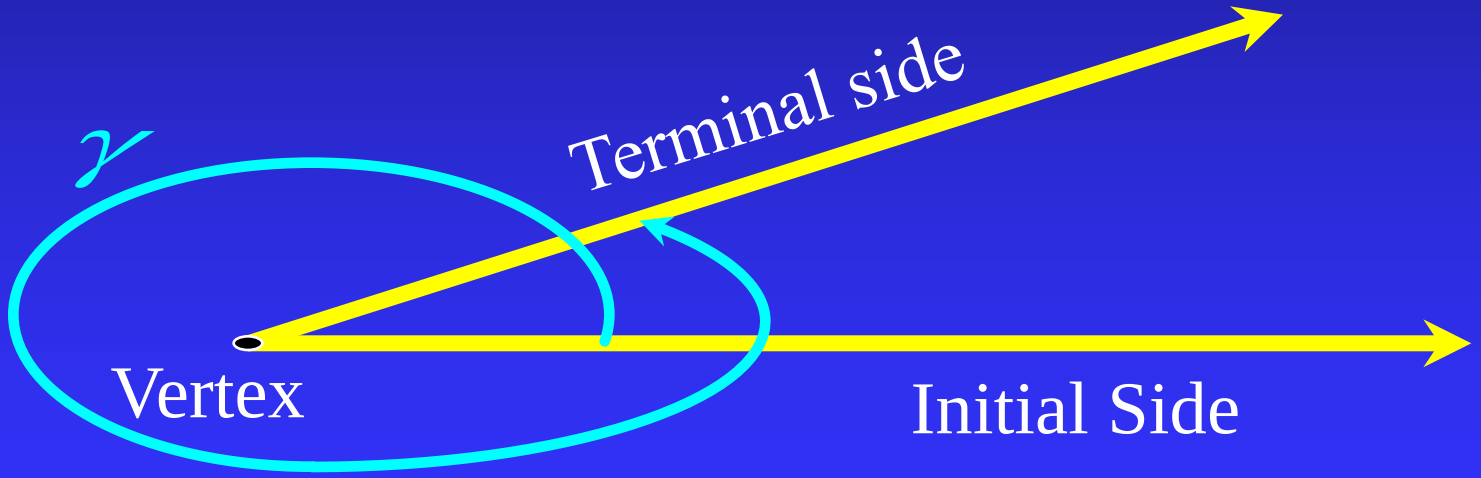
If two lines are drawn with a common vertex, they form an **angle**. One of the rays of an angle is called the **initial side** and the other the **terminal side**.



Counterclockwise rotation
Positive Angle

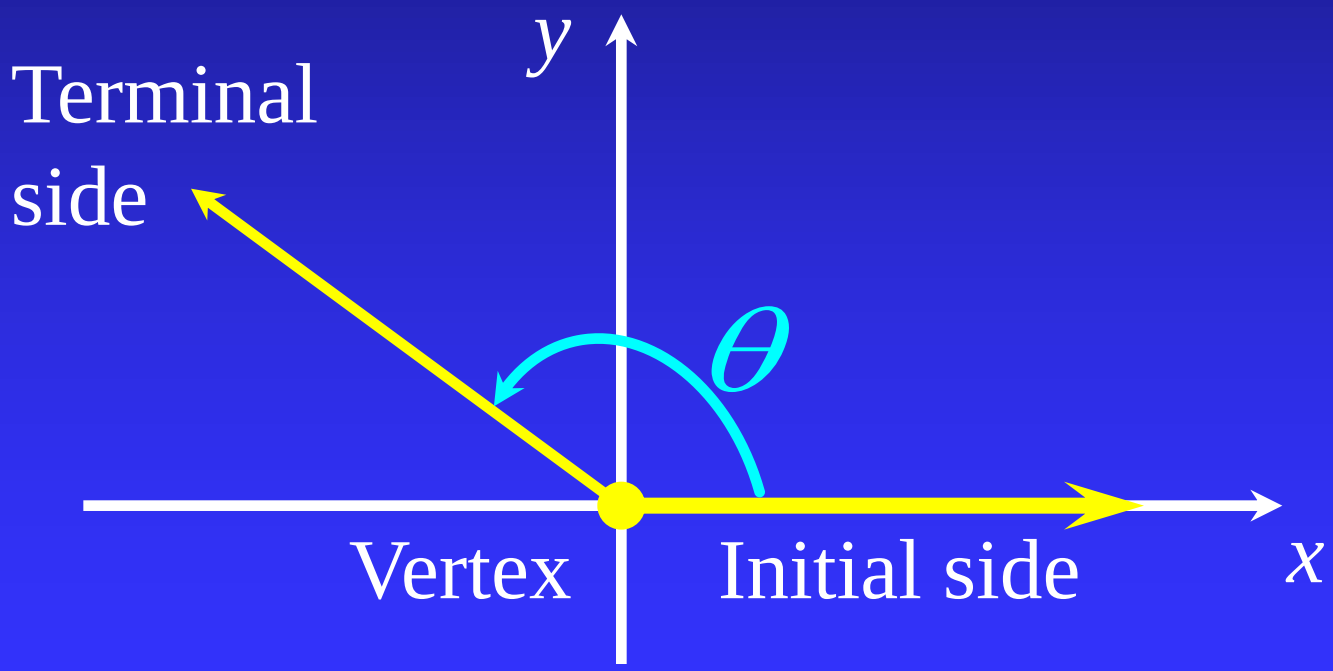


Clockwise rotation Negative Angle

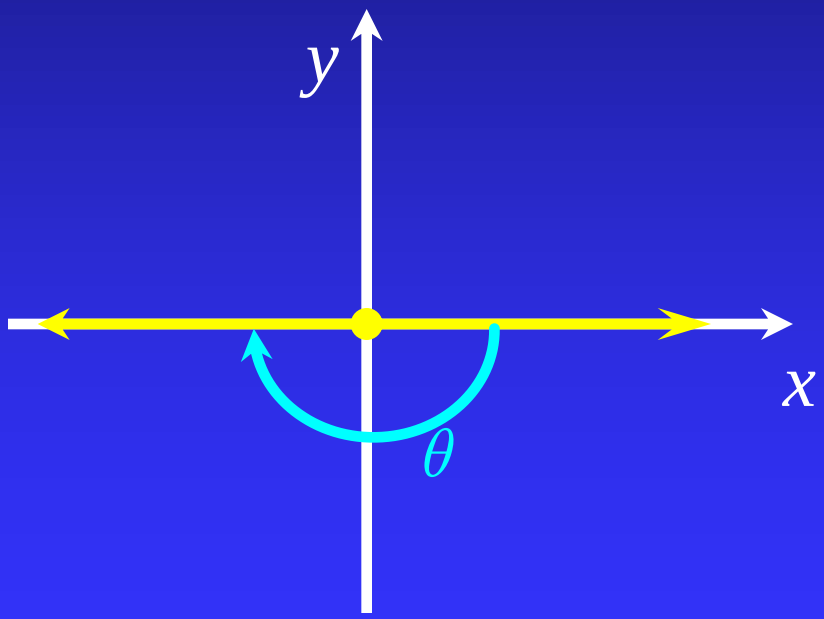


Counterclockwise rotation Positive Angle

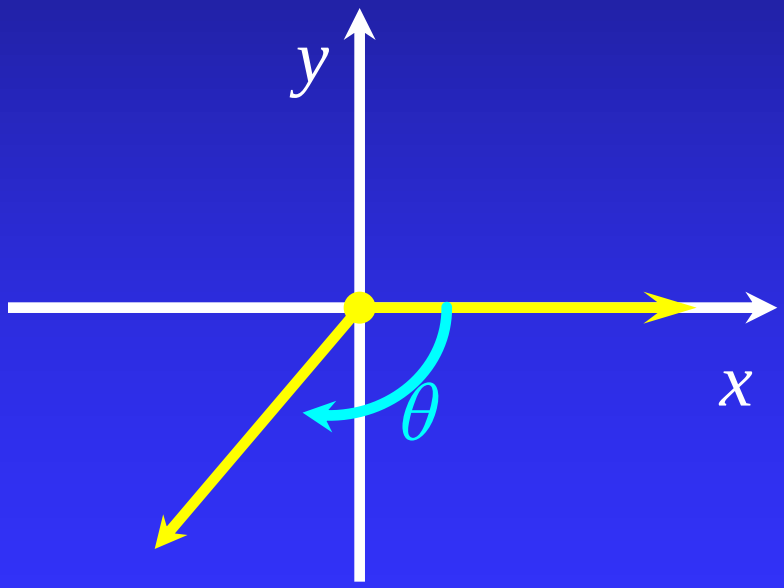
An angle θ is said to be in **standard position** if its vertex is at the origin of a rectangular coordinate system and its initial side coincides with with positive x-axis.



When an angle θ is in standard position, the terminal side either will lie in a quadrant, in which case we say θ **lies in that quadrant**, or it will lie on the x-axis or the y-axis, in which case we say θ is a **quadrantal angle.**



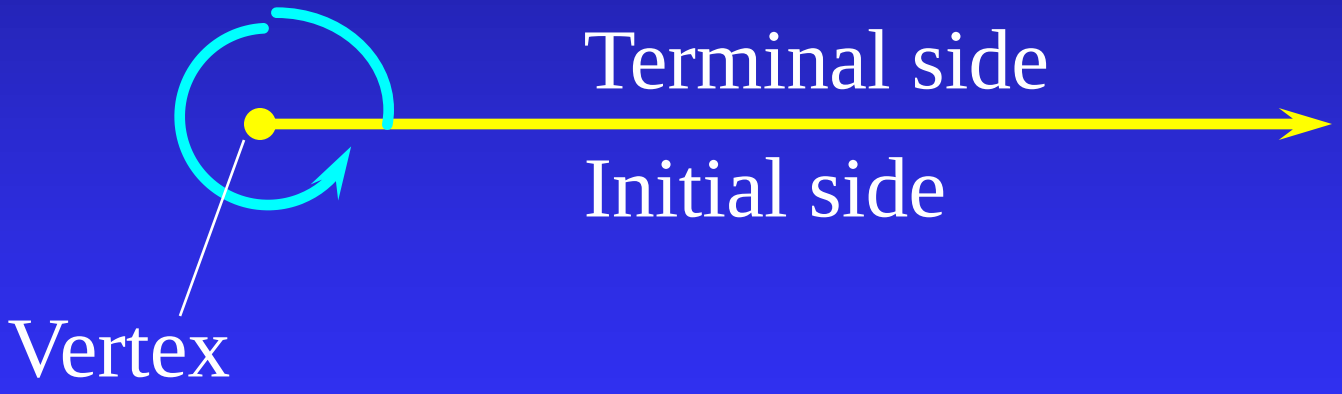
θ is a quadrantal angle



θ lies in Quadrant III

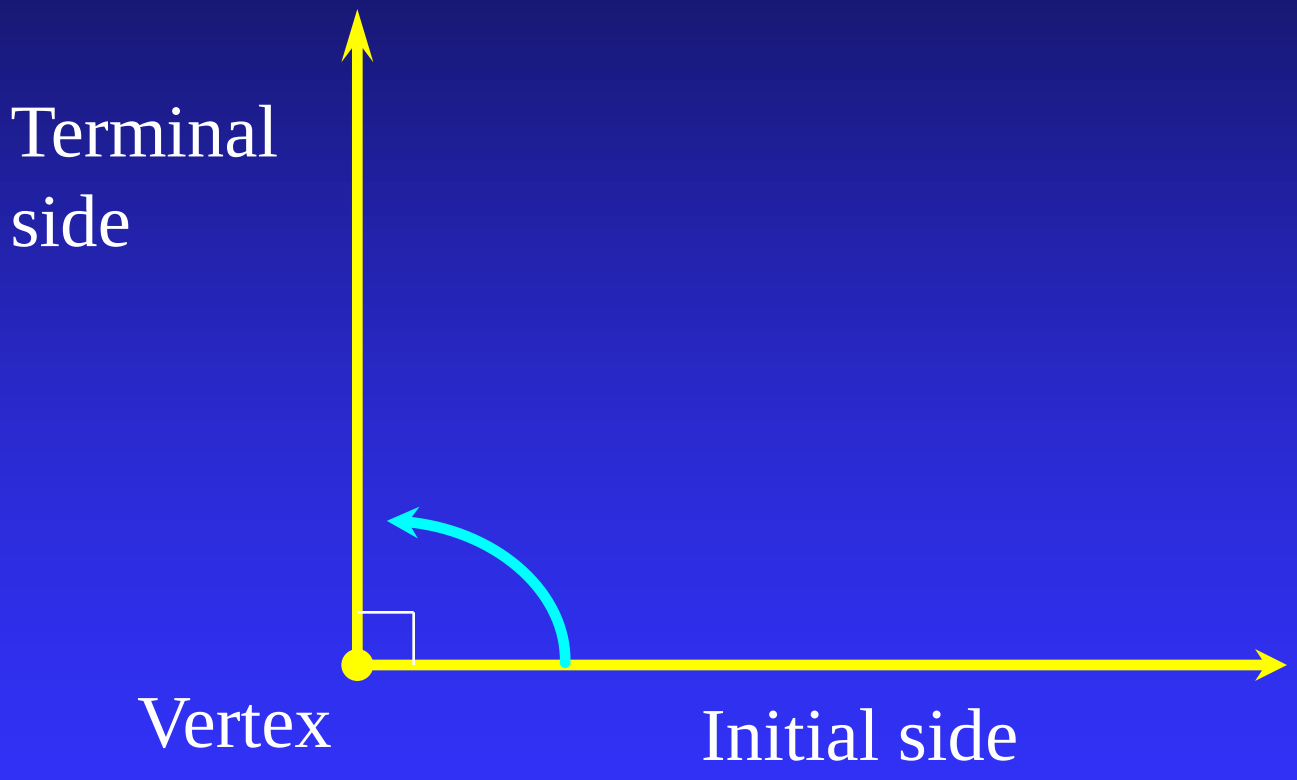
Angles are commonly measured in either Degrees or Radians

The angle formed by rotating the initial side exactly once in the counterclockwise direction until it coincides with itself (1 revolution) is said to measure 360 degrees, abbreviated 360°.



One degree, 1°, is $\frac{1}{360}$ revolution.

A **right angle** is an angle of 90°, or $\frac{1}{4}$ revolution.

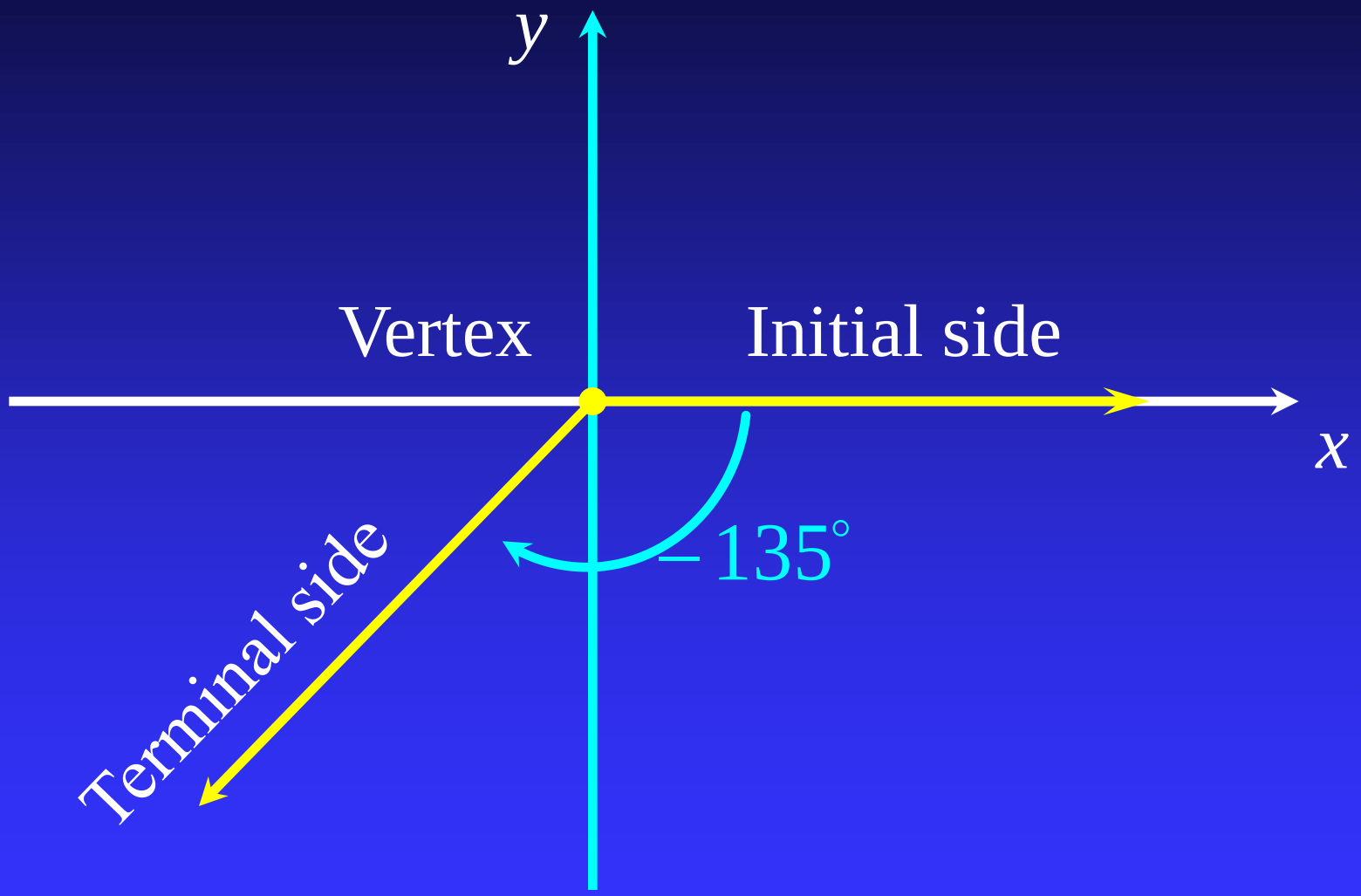


90° angle; $\frac{1}{4}$ revolution

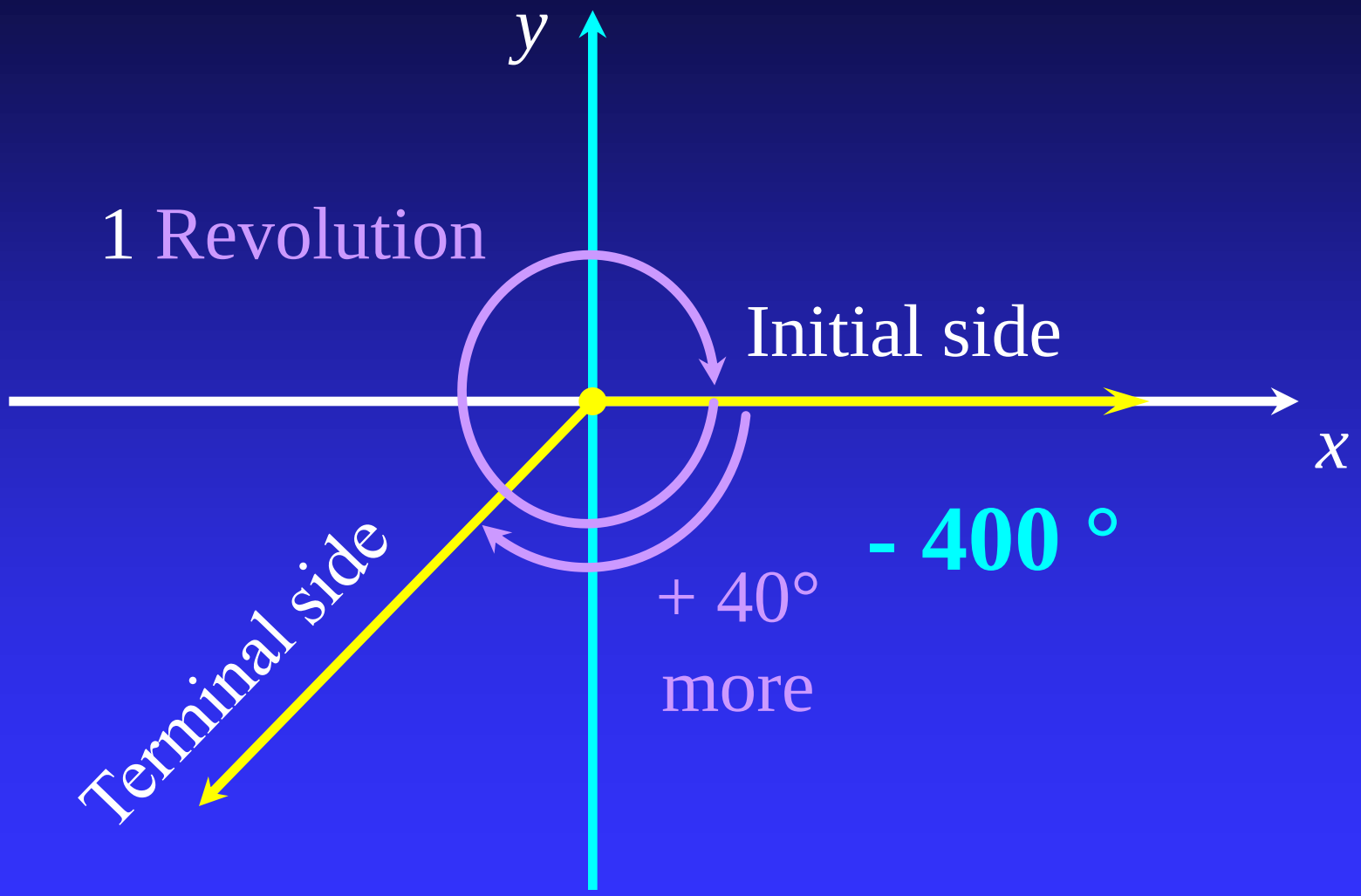
A **straight angle** is an angle of 180°, or $\frac{1}{2}$ revolution.

180° angle; $\frac{1}{2}$ revolution

Draw a -135° angle.



Because angles “rotate”, they can revolve around more than once . Shown here is a 400° angle.



Smaller Measures

- Angles measured in degrees is a standard unit of measure.
- When we need units smaller than degrees, they are called “Minutes” and “seconds”, modeled after our clock.
- 1 Revolution = 360° OR
  $1/360^{th}$ revolution = 1°

One minute, denoted, $1'$, is defined as $\frac{1}{60}$ degree.

One second, denoted, $1''$, is defined as $\frac{1}{60}$ second, or $\frac{1}{3600}$ degree.

1 counterclockwise revolution $= 360^\circ$

$60' = 1^\circ$ $60'' = 1'$

Sometimes we need to convert between decimal degrees (d.d.) and Degrees-Minutes-Seconds (D.M.S.)

Convert $30^\circ 12' 55''$ to a decimal in degrees.

$$30^\circ 12' 55'' = \left(30 + 12 \cdot \frac{1}{60} + 55 \cdot \frac{1}{3600}\right)^\circ$$

$$= (30 + 0.2 + 0.015278)^\circ$$

$$= 30.215278^\circ$$

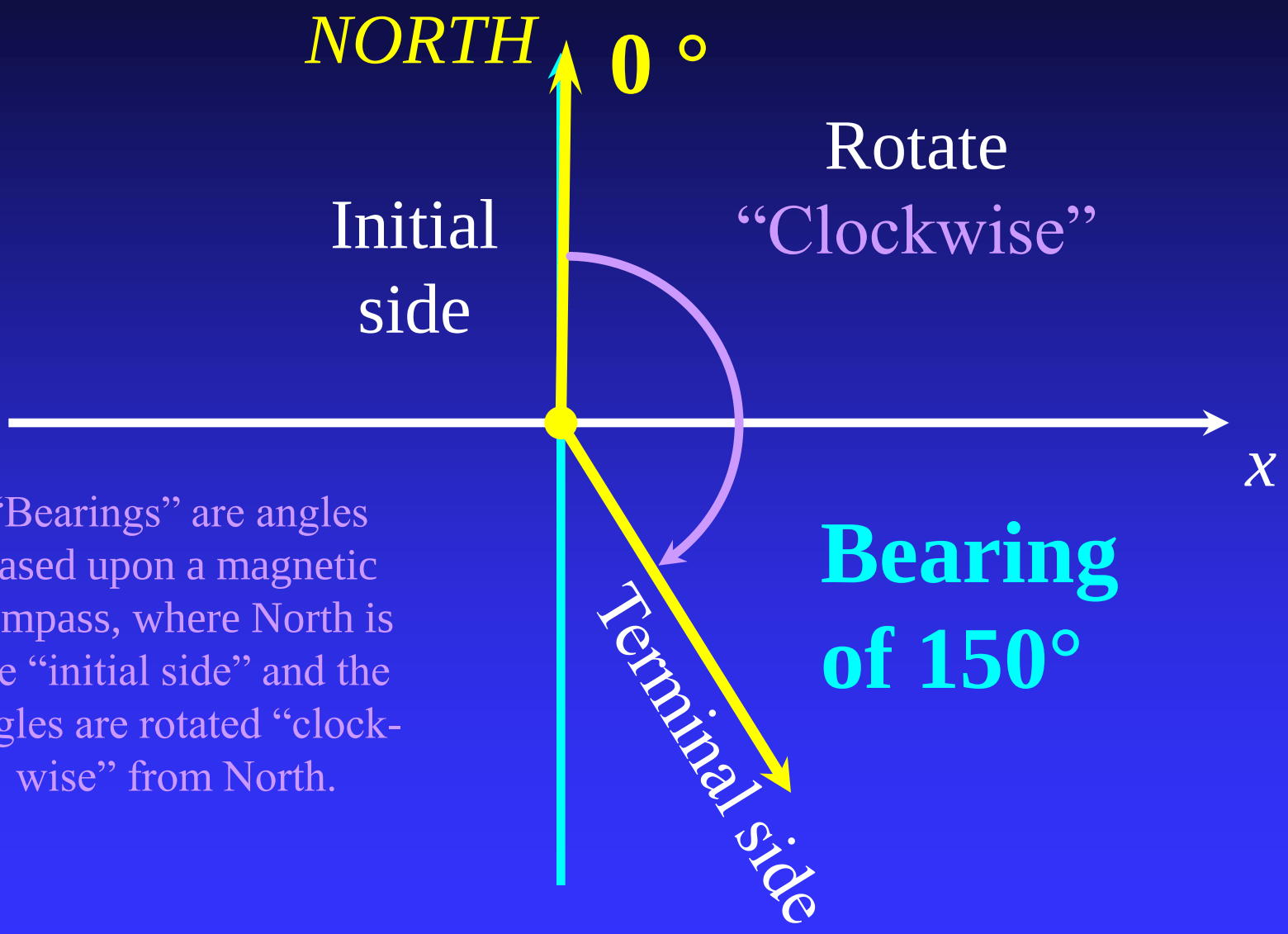
Convert 45.413° to D°M′S″ form.

$$0.413^\circ = 0.413^\circ \cdot \frac{60'}{1^\circ} = 24.78'$$

$$0.78' = 0.78' \cdot \frac{60''}{1'} = 46.8'' \approx 47''$$

$$45.413^\circ = 45^\circ 24' 47''$$

APPLICATION - NAVIGATION "Bearing"



"Bearings" are angles based upon a magnetic compass, where North is the "initial side" and the angles are rotated "clock-wise" from North.